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On the possible identity of Nova Geminorum with a small star photographed before the outburst. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. Soon after the announcement of the discovery of Nova *Geminorum*, I heard from Dr. Max Wolf, of Heidelberg, that he had taken a photograph of the region of the Nova on 1903 February 16. This plate is reproduced in *Monthly Notices*, vol. lxiii. plate 13 (opposite p. 330), though by an oversight the letters N and S, E and W, have been transposed; and Dr. Max Wolf remarked, "There is nothing exactly at the place of the Nova, but extremely near to it is a star-like object of about the sixteenth magnitude, and this extends over the place of the Nova, so that the Nova may be a part of this chain-like object."

Dr. Max Wolf kindly sent an enlargement of this negative to the University Observatory, Oxford, and we have measured the position of this object with reference to the stars immediately surrounding it, which have been connected with the Nova micrometrically by Professor Barnard, and also measured here on a plate of 100 min. exposure taken by Mr. Bellamy on April 22.

2. Later we heard that Mr. Parkhurst at the Yerkes Observatory had also photographed the region on February 21 with the 24-inch reflector and an exposure of twenty minutes, showing stars to the 15th or 16th magnitude. The purpose of the photograph was to obtain the region round *X Geminorum*, a variable discovered by Dr. Anderson. The position for 1900·0 of

	R.A. h m s	Decl. ° ′
X Geminorum is	6 40 43	+30 23·0
Nova *Geminorum*	6 37 49	+30 2·5

so that the stars near the latter are at the edge of Mr. Parkhurst's plate, and the images are fan-shaped. A positive copy of this plate was kindly sent by Mr. Parkhurst and received at Oxford on June 6. Measures of the same stars have been made on this positive, and also on a negative made from it.

3. The general result of all the measures is not favourable to the supposition that the faint star previously photographed is identical with the Nova, though it does not seem possible to decide the matter conclusively.

4. The stars near the Nova may be conveniently designated by the numbers assigned by Professor E. E. Barnard on p. 5 of *Bulletin* No. 19 of the Yerkes Observatory. His measures have been reduced to "Standard Co-ordinates" (with the Nova as centre) so that they may be compared more readily with measures on photographic plates. For a reason which will presently appear small constants (viz. those entered for n) have been added to all these co-ordinates to reduce them to the mean of the four stars 1, 4, 5, 6. These stars form an irregular quadrilateral enclosing the region of the Nova (n) and the stars 2 and 3; and any systematic errors for stars within this area ought to be smaller than those for the boundary. The columns headed "Oxford" give the differences from Barnard's results of Mr. Bellamy's measures on a plate of 100 min. exposure taken on April 22.

The unit is one réseau interval of 5′ or 300″.

Table I.

Standard Co-ordinates of Barnard's Stars and the corrections indicated by Oxford measures.

Star's No.	Barnard.		Oxford.	
	x	y	Δx	Δy
1	+·1332	−·0918	+·0004	·0000
4	−·0493	−·3371	+·0017	+·0017
6	+·1286	+·2637	−·0004	−·0020
5	−·2124	+·1653	−·0009	·0000
2	+·0577	−·2137	+·0021	+·0007
3	−·0303	+·1113	+·0061	−·0017
n	+·0285	−·0644	+·0027	−·0030

5. It will be seen that the agreement for the stars 2, 3, n (the Nova) is not very good, especially in the x co-ordinate. The differences cannot well be due to orientation or scale value, for they would show most in the outer stars. A source of error which has gradually been forced on our attention—what Kapteyn calls the "error of guiding"—seems also inadmissible here, since it produces an effect which increases systematically with the magnitude; and though the stars 2 and 3 are all fainter than 1, 4, 5, 6, and their positive Δx might be due to an "error of guid-

ing," the Nova is of course much brighter and should have a negative residual on this supposition. It seems difficult to account for these errors except as accidental; and, if this is the true view of them, it is worth noting before we pass to the other photographs which have defects from which the Oxford plates ought to be free; viz. they are on a smaller scale, and in the case of Mr. Parkhurst's photograph the images are near the edge of the plate and consequently fan-shaped.

6. Dr. Max Wolf's enlargement was measured both by Mr. Bellamy and myself on April 20, being placed film to film with a réseau for the purpose of measurement. For scale value and orientation five stars were measured, surrounding the region of the Nova and at some distance from it; and it was inferred that measures (x_w, y_w) on the Max Wolf plate were connected with standard co-ordinates (ξ, η) by the approximate formulæ

$$\xi = 0{\cdot}800\ x_w - {\cdot}011\ y_w + \text{const.}$$
$$\eta = 0{\cdot}800\ y_w + {\cdot}011\ x_w + \text{const.}$$

7. It was considered that these coefficients were probably sufficiently exact for application over the small region under examination, the constants being determined from the four stars 1, 4, 5, 6. The images on the enlargement were of very irregular shape, and a more exact determination of the coefficients and constants would require an elaborate discussion of the plate. It may be ultimately necessary to undertake this, but the present preliminary inquiry will at any rate suffice to indicate the limits of possible uncertainty.

8. The Parkhurst plate was measured by Mr. B. Gray and myself on June 11, being first copied on to a plate with a réseau on it. Measures of four stars in different parts of the plate gave for the approximate constants of reduction

$$\xi = -\ 1{\cdot}462\ x_P - {\cdot}0092\ y_P + \text{const.}$$
$$\eta = -\ 1{\cdot}462\ y_P + {\cdot}0092\ x_P + \text{const.}$$

but if these formulæ are applied to the whole plate, the residuals for different stars vary considerably. For the region of the Nova, the co-ordinates of which are about $x_P = 15{\cdot}7$ $y_P = 9{\cdot}6$, the constants differ by about $+\ 0{\cdot}030$ in ξ and $+{\cdot}010$ in η from those deduced from the four reference stars, the co-ordinates of which are (10·7, 4·0) (14·2, 2·4) (14·1, 11·2) (11·5, 10·4). It will be seen that these are all on one side of the Nova, which is too near the edge of the plate to allow of reference stars on the other side. It is more than ever necessary to determine the constants from the stars 1, 4, 5, 6, which surround the region under consideration. Mr. Plummer has tried to find some satisfactory method of dealing with plates taken with reflectors, but without much success; perhaps we may hope for better success in the future; but meanwhile we must fall back on the expedient before used,

R R 2

of trusting an approximate scale value and orientation over the small area, and determining the constants from the four corner stars.

9. I proceed to give the "Corrections to Barnard" indicated by the three photographs. It seems useless to go further than ·001 of a réseau interval, in view of the great uncertainties of the measures. The objects which might be the Nova on the first two plates have been referred to Barnard's position of the Nova as given in Table I.

TABLE II.

Star's No.	Max Wolf, Feb. 16.		Parkhurst, Feb. 19.		Oxford.	
	Δx	Δy	Δx	Δy	Δx	Δy
1	+·001	−·005	−·002	+·001	·000	·000
4	−·005	+·002	+·002	+·003	+·002	+·002
6	+·001	+·004	·000	−·002	−·001	−·002
5	+·002	−·002	·000	−·002	−·001	·000
2	−·001	+·013	−·002	−·001	+·002	+·001
3	−·015	−·002	−·006	−·005	+·006	−·002
n	−·016	−·009	−·021	+·005	+·003	−·003
n'	−·070	−·012	—	—	—	—

10. These figures give us some idea of the limits of error. Star 2 has a residual of +·013 or +4″ in y, which is not supported either by the Parkhurst or the Oxford plate, and must be regarded as accidental. The Max Wolf plate has been magnified five times, and the images are large and irregular. The x measures for star 3 are somewhat remarkable. Both plates taken before the outburst indicate a negative correction to Barnard's measures of March 30, and the Oxford plate, taken on April 22, shows a distinctly positive correction. The figures would be suited by the supposition of a very rapid proper motion (20″ a year say), but this supposition is scarcely admissible, at any rate without further evidence. I wrote to Professor Barnard, hoping that he might have measures on different nights which would give some information. The measures he made were as follows :

Nova and Star 3.

1903 March 30	341°·61	55″·69
April 27	341°·45	(too difficult)

Since a P.M. in R.A. would largely affect the position-angle, these measures are good evidence against any exceptional proper motion.

11. We now come to the question whether the Nova is identical with the small star near its place. The place of the Nova itself seems to be satisfactory, the Oxford measures being in fair agreement with Professor Barnard's. On the Max Wolf plate the image near the place of the Nova is double; in the letter quoted in the first paragraph it is called a "chain-like object." The Parkhurst plate does not support the view that there is a double star at this place, and the double appearance is probably a photographic defect. If it is an elongated image of a single star, we should take some sort of a mean between the positions given for the two condensations n and n'; but the most favourable supposition for identity with the Nova is clearly that n' is spurious, and n is the true image of the star shown on the Parkhurst plate. Even then it differs from the Parkhurst object by ·014 or 4″ in y, in the opposite direction to the difference for star 2. In the light of these discordances, are we to regard the differences from Barnard in the x co-ordinate as merely accidental? If the Max Wolf plate had stood alone, the residuals for stars 2 and 3 would certainly suggest that those for the star n might be accidental; but there is a striking accordance between the x residuals for the two plates which makes one hesitate to believe that the star n can be really the Nova, though this is far from impossible.

12. A word may be said as to the method of measurement. For the Max Wolf plate we simply bisected, as well as possible, the large and irregular images formed. The grain of the film is so large in proportion to them that it seems doubtful whether any systematic errors could be determined. But in the case of the Parkhurst plate, which was taken with a *reflector*, it becomes important to know how far the known systematic errors of reflector photographs could remove or diminish the residuals on a favourable supposition. Reflector images away from the plate centre are fan-shaped, and the apex of the fan corresponds to the ray reflected from the centre of the mirror. If we could measure the exact apex, we should get a true geometrical projection on the plate. Unfortunately the apex is in general cut off by the small mirror (or by the plate-holder itself), and cannot be measured. It is even difficult to estimate its position, owing to the photographic encroachment round the edges of the geometrical image. Mr. Plummer has suggested measuring the transverse axis of the fan, but the results of a trial of this method were not very satisfactory, and we seem almost driven to empirical rather than theoretical methods of measurement. But we may form a fair idea of the possible amount of error by measuring in different ways. The fan-shaped images on the Parkhurst plate were measured by each of two observers in two ways:

(*a*) Estimating the apex of the fan as well as possible.

(*b*) Estimating the centre of the transverse axis as well as possible.

The differences are as below. Of course, if they were constant throughout they would not affect the question at all.

TABLE III.

Star's No.	x Diff. H.T.	x Diff. B.G.	y Diff. H.T.	y Diff. B.G.
1	+012	+013	+008	+009
4	+009	+012	+008	+009
6	+015	+013	+006	+010
5	+012	+013	+004	+010
2	+006	+006	+004	+007
3	+008	+009	+010	+006
n	+010	+014	+008	+010

It will be clear that we cannot look to any great alteration of the residuals from methods of measurement.

13. On the whole, the evidence seems to be unfavourable to the view that the Nova is identical with the small star photographed before the outburst; but the question cannot be regarded as settled until the light of the Nova has faded sufficiently for search to be made for a small star some $5''$ away. For the present it seems best to publish these figures without attempting to draw any final conclusion. But measures of these stars, especially of star 3, will be of great interest until the questions raised are settled.

University Observatory, Oxford:
1903 *August* 12.

P.S. September 14.—The printers' proof of the above was sent back to them yesterday. I have this morning received a letter from Professor Barnard which seems to show that the object photographed on February 16 and 19 is *not* identical with the Nova. He says, under date September 1, "This morning I got your Nova in *Gemini*. There is a small star 14^m preceding it which I measured, and which seems to be Mr. Parkhurst's star:

Position-angle $284^\circ{\cdot}3$ Distance $7''{\cdot}83$.

I shall measure it again. The distances were measured under difficulties of approaching day, but I do not think the result can be more than $\frac{1}{2}''$ out. The Nova has got down to between the 11th and 12th magnitudes. It appeared hazy to me, like a hazy nebulous star. The seeing was good. It looks like Nova *Persei*, but I think is fainter."

A Derivation of Hill's Equation by a Direct Substitution.
By R. A. Herman, M.A.

(*Communicated by the Secretaries.*)

The method and most of the notation is that of Hill's paper on the "Motion of the Lunar Perigee," *Acta Mathematica*, vol. viii.

The equations satisfied by u and s are

$$\left.\begin{aligned} D^2u + 2mDu + 2\frac{\partial \Omega}{\partial s} &= 0 \\ D^2s - 2mDs + 2\frac{\partial \Omega}{\partial u} &= 0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad (1)$$

which lead to Jacobi's Integral

$$DuDs + 2\Omega = 2C \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

We have if ψ be the angle made by the tangent with the axis of x

$$Du = \zeta\frac{d}{d\zeta}(x+is) = e^{i\psi}\zeta\frac{d\sigma}{d\zeta}, \quad Ds = e^{-i\psi}\zeta\frac{d\sigma}{d\zeta}$$

where σ is the arc of the curve; or if ϖ denote $Du \,.\, Ds$,

$$Du = e^{i\psi}\sqrt{\varpi}, \quad Ds = e^{-i\psi}\sqrt{\varpi}$$

Hence $2i\psi = \log(Du/Ds)$; and

$$2iD\psi = \frac{D^2u}{Du} - \frac{D^2s}{Ds} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

This gives $2i\varpi D\psi = D^2uDs - D^2sDu$, and on substitution from (1)

$$\begin{aligned} 2i\varpi D\psi &= -Ds\{2mDu + 2\Omega_s\} - Du\{2mDs - 2\Omega_u\} \\ &= -4m\varpi - 2\Omega_sDs + 2\Omega_uDu \end{aligned}$$

Hence
$$\Omega_uDu - \Omega_sDs = \varpi(2m + iD\psi) \quad \ldots \quad \ldots \quad (4)$$

Differentiating (4)

$$\Omega_uD^2u - \Omega_sD^2s + (\Omega_{uu}Du + \Omega_{us}Ds)Du - (\Omega_{us}Du + \Omega_{ss}Ds)Ds = (2m + iD\psi)D\varpi + \varpi iD^2\psi$$

or substituting for Ω_u and Ω_s from (1)

$$m(DuD^2s + DsD^2u) + \Omega_{uu}\overline{Du}|^2 - \Omega_{ss}\overline{Ds}|^2 = (2m + iD\psi)D\varpi + \varpi iD^2\psi$$

giving the equation

$$(i\varpi D^2\psi + \Omega_{ss}\overline{Ds}|^2 - \Omega_{uu}\overline{Du}|^2) = -(m + iD\psi)D\varpi \quad \ldots \quad (5)$$

which will be useful later.